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On the Long-Tail Solar Wind Electron Velocity Distribution

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1. INTRODUCTION

Like most winds,[1] the solar wind has a large degree of variability. However, even extremely erratic events can have predictable features, if these events are governed by an underlying probability distribution. In this paper we stress the role of the log-normal distribution in the description of the high-energy tail of the electron velocity distribution in the solar wind plasma and the relationship of this distribution to a Maxwellian (Gaussian) distribution at lower velocities. Howard Reiss has devised masterful uses of probability limit distributions in several areas of science, including in theories of nucleation, magnetic relaxation, and even traffic flow. It is an honor and a pleasure to dedicate this paper to him on the occasion of his 66th birthday.

2. FROM KOLMOGOROV'S ROCKS TO SOLAR ELECTRONS

In 1941 Kolmogorov[2] considered the question of the distribution of crushed ore sizes. His major assumption way that if a rock is of scale size R_0 initially, then it is of scale size $R_1 = \lambda_1 R_0$ after one break, where λ_1 is a random number uniformly distributed between zero and unity. After a

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succession of breaks the rock has the scale sizes R_1, R_2,..., R_N, where the difference $R_{n-1} - R_n$ satisfies

$$R_{n-1} - R_n = (1 - \lambda_n) R_{n-1} \tag{1}$$

(Note $R_{n-1} > R_n$ and $0 < \lambda_n < 1$.)

Setting $1 - \lambda_n = q_n$, one chooses the following combination of random variables:

$$\sum_{n=1}^{N} \frac{R_{n-1} - R_n}{R_{n-1}} = \sum_{n=1}^{N} q_n \tag{2}$$

to form (in the continuum limit with $dR = R_{n-1} - R_n$) the following integral:

$$\int_{R_N}^{R_0} \frac{dR}{R} = \ln \frac{R_0}{R_N} = q_1 + \cdots + q_N \tag{3}$$

The sum of the λ_i has a Gaussian distribution by the central limit theorem, which implies that R_0/R_N has a log-normal distribution. Note that R_N can also be written as a

$$R_N = \prod_{n=1}^{N} \lambda_n R_0 \tag{4}$$

which directly gives the result that R_N has a log-normal distribution,

$$f(R) = (2\pi\sigma^2)^{1/2} R^{-1} \exp[-(\ln R/\langle R \rangle)^2/2\sigma^2] \tag{5}$$

where $\langle R \rangle$ and σ^2 are the average and variance of the distribution. Note that in Eq. (5) we have ignored the fact that the maximum value of R is R_0.

Charged particle velocity distributions in the solar wind and the magnetospheres of the planets usually deviate markedly at high velocities from a Maxwellian (Gaussian) distribution. This deviation is often described in terms of a "core" and "halo" distribution for solar wind electrons.[3] The "core" at low to moderate velocities is well represented by a Maxwellian distribution, while the high-velocity "halo" has an amplitude much greater than predicted by the projected core distribution. An example of this core–halo distribution is given in Fig. 1, where a typical solar wind electron velocity distribution (see Feldman *et al.*[3]) is plotted as a function of reduced velocity, defined as velocity divided by mean square deviation of the velocity distribution. The data show a transition from a Maxwellian

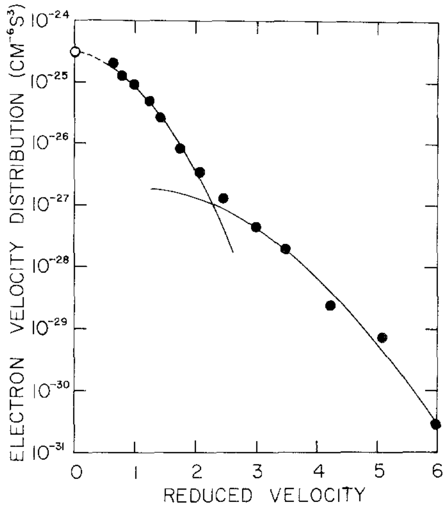

Fig. 1. Velocity distribution function for solar wind electrons as a function of the reduced velocity as defined in the text. The lines through the data emphasize the core–halo structure. The data point at zero reduced velocity is an extrapolation of the higher velocity data. The data are those of Feldman *et al.*[3]

"core" distribution at velocities below 2 to a more slowly varying halo function at higher velocities. The two lines drawn through the data emphasize this fact.

Detailed microscopic models for the motion of the electrons in the solar wind have been used to explain the core–halo distribution.[4] In general, the models rely on the facts that the electron–electron collision cross section decreases as $1/v^4$, where v is the relative electron velocity, and that the electron density decreases as $1/r^2$, where r is the heliocentric radius.

In this paper we show that the core–halo solar wind velocity distribution function can be understood in terms of a simple phenomenological model of general applicability in which the core has a Maxwellian or normal distribution and the halo a log-normal distribution. Furthermore, in the presene of structures in the interplanetary medium capable of interacting with the electrons, the model predicts a transition at the highest velocities to a secondary halo distribution.

The assumption we employ is that solar electrons are produced with a distribution with a high-velocity component. It is most likely that the higher velocity electrons collide with slower moving electrons, since the scattering cross section varies inversely with the fourth power of velocity and the number of low-velocity electrons far exceeds those at high velocities. The high-velocity electrons will lose energy to the lower energy electrons in an amount depending upon their relative velocities and impact parameter. We assume that the initial magnitude of velocity $|\mathbf{v}_0|$ of a solar electron is reduced to $\lambda_1 |\mathbf{v}_0|$ $(0 < \lambda_1 < 1)$ after a single collision, and that several collisions occur. After n collisions the velocity is $\lambda_1 \cdots \lambda_n |v_0|$ (with, as before, λ_i a uniform random variable taking values between zero and unity). In complete analogy with Kolmogorov's problem, this analysis leads us directly to a log-normal distribution $f(\mathbf{v})$ for the distribution of electron velocities.

3. A LOW-ENERGY CROSSOVER

The log-normal distribution cannot hold for all values of $|\mathbf{v}|$ because after several collision $|\mathbf{v}|$ is reduced to a sufficiently low velocity that the solar electron is just as likely to gain as to lose energy in a collision. This is the condition for thermalization and a Maxwellian or Gaussian distribution must govern the electron velocity distribution at low velocities.

To show the distinction between the low- and high-velocity distributions more clearly, we have plotted the experimental cumulative distribution function using the data of Fig. 1 on normal and log-normal probability paper[5] in Figs. 2a and 2b. The only difference between the

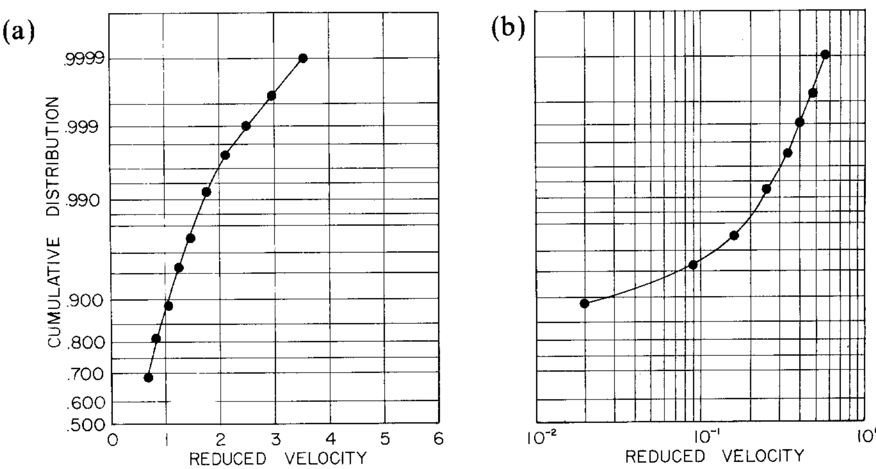

Fig. 2. (a) Cumulative velocity distribution function for the data in Fig. 1 plotted on normal probability paper. The scales are arranged so that a normal distribution function will lie on a straight line. (b) Cumulative velocity distribution function for the data in Fig. 1 plotted on log-normal probability paper. The scales are arranged so that a log-normal distribution function will lie on a straight line.

figures is the scale of the independent variable. A normal distribution appears as a straight line on normal paper (Fig. 2a) and a log-normal distribution will give a straight line on log-normal paper (Fig. 2b). It is clear that the normal–log-normal representation of the data does indeed provide a reasonable fit to the experimental points.

4. A HIGH-ENERGY CROSSOVER

Let $G(v)$ be the velocity distribution of solar wind electrons and let $g(v)$ be the log-normal distribution generated from electron–electron collisions. If $G(v) = g(v)$, then only the previously discussed electron–electron collisions determine the form of $G(v)$. Let us suppose other processes occur which affect the electron velocity. One candidate is the interaction of electrons with a hierarchical interplanetary magnetic field structure as discussed by Matthaeus and Goldstein.[6]

To begin our discussion, let us denote the average value of $g(v)$ by $\langle v \rangle$. Assume there are interplanetary medium structures of an unspecified nature with which the electrons can collide to produce a new log-normal distribution $g_1(v) = (1/s)\, g(v/s)$ $(s > 1)$. Note that the mean value of $g_1(v)$ is $s\langle v \rangle$. If a fraction γ of the electrons encounter one of these structures and a fraction γ of these encounter two structures and so on, then

$$G(v) = (1-\gamma)\left[g(v) + \frac{\gamma}{s}\, g\left(\frac{v}{s}\right) + \frac{\gamma^2}{s^2}\, g\left(\frac{v}{s^2}\right) + \cdots \right] \tag{6}$$

In our initial discussion $\gamma = 0$, so only the first log-normal term entered into the discussion.

Note that $G(v)$ satisfies the scaling equation[7]

$$G(v) = \frac{\gamma}{s}\, G\left(\frac{v}{s}\right) + (1-\gamma)\, g(v) \tag{7}$$

While the solution for $G(v)$ resembles $g(v)$ for small v, it differs substantially as v increases. The asymptotic behavior of $G(v)$ is

$$\lim_{v \to \infty} G(v) = v^{-1-\mu}$$

where

$$\mu = -\log \gamma / \log s$$

Thus, we expect $G(v)$ to be Maxwellian for small v, log-normal for intermediate v, and algebraically decaying for large v.

Electron velocity distribution data are available at energies up to MeV; however, at energies above approximately 1 keV (reduced velocity of 10) other processes originating in the solar corona give rise to electrons that effectively mask these interactions.[8] Were it possible to separate the high-energy coronal electrons from the solar wind electrons, the parameter μ could be obtained from the slope of the high-velocity portion (above 1 keV) of the distribution. The parameter s could be derived from μ and with it the hypothesized interplanetary structures.

In conclusion, we emphasize that although the above statistical approach to understanding velocity distribution functions says very little about microscopic plasma processes, it does provide a framework for a discussion of the general classes of interactions that exist.

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